A-Level Mathematics – Summer Preparation Pack

Introduction

A-Level mathematics is a very rich and rewarding course, that is extremely well valued by both employers and universities. It is, however, a challenging course. We want you to be ready to face these challenges and to do so you must be fluent in the number and algebraic skills that you studied at GCSE.

The purpose of this booklet is to inform you of the required skills for the A-Level mathematics course and the resources available to you to help you prepare.

How to Prepare

During the first couple of weeks of the course you will take a diagnostic test to assess these key number and algebraic skills. If you score less than 50%, you will be required to join additional bridging classes, in which you will continue to work on these skills alongside the new A-Level content. A sample diagnostic test is included in this booklet to help you prepare.

A useful approach to your preparation is to do the sample diagnostic test in test conditions, without the help of textbooks or additional resources. Mark your test to identify the areas that you need to work on. You can then focus your preparation on these areas, using the resources that are detailed in this booklet.

Recommended Resources

We strongly recommend purchasing and using the book by CGP "Head Start to A-Level Maths", ISBN 978-1782947922. This short book provides practice for all the key skills assessed in the diagnostic test and is available from Amazon (and other retailers) for approximately £6. You will see that the sample diagnostic test is taken from this book and the sections of the test link to the pages of the book that will support you in the corresponding areas.

Alternative resources for practice include Sparx maths and the A-Level revision website (https://alevelmathsrevision.com/bridging-the-gap/). Both these excellent resources provide explanatory videos as well as plenty of practice questions. Note that the list of Sparx tasks is extensive as it covers all expects of the listed topics – select the tasks that are appropriate for your level.

The table in the 'Required Skills' section of this document details each of the required skills, along with details of how you can practise these skills from the recommended resources.

Types of Number and Fractions

These topics are covered in Section 1 — p.6-7.

1) Which of the following are integers?

   4 -3.5 0.3 $\frac{4}{5}$ 8.99 -10 205 0

2) Which of the following values are rational, and which are irrational?

   $5.\dot{9}$ π $\sqrt{7}$ $\frac{1}{5}$ -6 $\sqrt{4}$ 13.978 2.1

3) Evaluate the following without using a calculator, giving your answers in their lowest terms. Give any answers larger than 1 as improper fractions.

   a) $\frac{2}{9} \times \frac{3}{5}$ b) $\frac{1}{6} \div \frac{2}{3}$ c) $\frac{1}{12} + \frac{5}{6}$ d) $\frac{8}{5} - \frac{1}{7}$

Indices, Multiplying Out Brackets and Factorising

These topics are covered on p.8-11.

4) Simplify the following:

   a) $x^7 \times x^2$ b) $10y^3 \div 5y$ c) m^0 d) $(2n^2)^5$

5) Write 5^{-2} as a fraction.

6) Evaluate the following without using a calculator:

   a) $\left(\frac{3}{4}\right)^2$ b) $16^{\frac{1}{2}}$ c) $8^{\frac{2}{3}}$ d) $36^{-\frac{1}{2}}$

7) Multiply out the brackets and simplify your answers where possible.

   a) $(x + 4)(x - 6)$ b) $(x + 5)^2$ c) $(2x - 1)(x + 3)$ d) $(x + 1)(x - 4)(x + 5)$

8) Factorise the following:

   a) $5x + 20$ b) $3a + 12ab$ c) $x^2 - 4$ d) $9x^2 - 36$ e) $x^2 - 5$

Surds

This topic is covered on p.12-13.

9) Simplify the following:

   a) $\sqrt{3} \times \sqrt{2}$ b) $(\sqrt{5})^2$ c) $\frac{\sqrt{30}}{\sqrt{6}}$ d) $\sqrt{12} + 2\sqrt{3}$ e) $(1 + \sqrt{7})^2$

10) Rationalise the denominators of the following:

   a) $\frac{3}{\sqrt{2}}$ b) $\frac{\sqrt{5}}{2\sqrt{2}}$ c) $\frac{2}{3 + \sqrt{6}}$ d) $\frac{\sqrt{2}}{1 - \sqrt{5}}$

Solving Equations and Rearranging Formulas

You'll find these on p.14-15.

11) Solve the following:

   a) $5x - 2 = 8$ b) $3(x - 6) = 2(x - 4)$ c) $\frac{x+2}{3} + \frac{2x}{5} = x + 2$ d) $2x(x + 1) = 2x + 18$

12) Make x the subject of the following formulas:

   a) $y = mx + c$ b) $y = \frac{3x + 2}{5}$ c) $y = 2x^2z + 1$ d) $y = \frac{3x + 1}{x - 2}$

Quadratic Equations

Quadratics are covered in Section 3 — p.16-21.

13) Solve the following by factorising:

   a) $x^2 - 3x + 2 = 0$ b) $x^2 + 6x + 5 = 0$ c) $2x^2 - 3x - 5 = 0$ d) $3x^2 - 13x = -12$

14) Solve the following using the quadratic formula.
    Give your answers to two decimal places.

   a) $x^2 + 2x - 10 = 0$ b) $2x^2 - 5x - 1 = 0$

The formula is: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

15) Solve the following by completing the square. Give your answers as surds.

   a) $x^2 - 4x - 2 = 0$ b) $2x^2 + 4x - 7 = 0$

16) a) Complete the square for $x^2 + 6x + 8$.

   b) Hence sketch the graph of $y = x^2 + 6x + 8$, labelling
      the turning point and intercepts with the x-axis.

Algebraic Fractions, Inequalities and Simultaneous Equations

17) Simplify the following:

   a) $\frac{15a^3b^3}{5a^2b}$ b) $\frac{2x^2y}{(4xy)^2}$ c) $\frac{x^2 - 16}{x^2 - x - 20}$

These topics are on p.22-29.

18) Simplify the following:

   a) $\frac{9b^2}{a} \times \frac{2a^2}{3b}$ b) $\frac{2(x-1)^2}{15} \times \frac{10}{4x - 4}$ c) $\frac{3x^2 - 21x}{x + 2} \div \frac{x(x-7)}{9x + 18}$ d) $\frac{3}{x + 1} + \frac{2x - 3}{x^2}$

19) Solve the following inequalities:

   a) $7x + 5 \le 2x$ b) $2(10 - x) > 4$ c) $2x^2 + 3 < 21$
   d) $4x^2 - 9 \ge 7$ e) $x^2 - 4x + 10 \ge 2x + 5$

20) Draw a set of axes with the x-axis from -2 to 3 and the y-axis from 0 to 6.
    Show on these axes the region that satisfies the following inequalities:

   $$y > 3x - 1, \qquad y < x + 3 \qquad \text{and} \qquad y \ge \frac{x}{5} + 2$$

21) Solve the following simultaneous equations:

   a) $2x + y = 2$
      $x - 3y = 8$
   b) $3x - 2y = 1$
      $5x - 3y = 7$
   c) $y = x^2 + 3$
 $y - 2x = 18$
   d) $3y = 2(x^2 - 3)$
      $2x - y = 2$

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Proof and Functions

22) Prove that the sum of any three consecutive odd numbers is a multiple of 3.

23) Naveen says, "for any integers x and y, $xy > y$". Prove that Naveen is wrong.

24) $f(x) = \dfrac{x+5}{3}$ and $g(x) = x - 3$.

    a) Evaluate $f(4)$.
    b) Find $fg(x)$.
    c) Find $f^{-1}(x)$.

Straight Lines and Quadratic Graphs

25) Give the gradient and y-intercept of the line $x + 2y = 4$.

26) Point A has coordinates $(5, 2)$ and point B has coordinates $(2, -4)$.

    a) Find the equation of the line passing through points A and B.
    b) Find the exact length of line AB.

27) Line A has equation $y = 2x + 5$.

    a) Find the equation of the line parallel to line A which passes through $(3, 2)$.
    b) Find the equation of the line perpendicular to line A which passes through $(2, 1)$.

28) Sketch the graph of $y = x^2 - 8x + 15$. Label the graph with the coordinates of the turning point and the points where the graph crosses the axes.

Harder Graphs and Graph Transformations

29) Sketch the following graphs:

    a) $y = x^3$
    b) $y = \dfrac{1}{x}$
    c) $y = -\dfrac{1}{x}$

30) The graph on the right shows how the number of fish (F) living in a river changes over time. The equation of the graph is $F = mn^t$ where t is the number of years and m and n are positive constants. Find the values of m and n.

31) Find the equation of the tangent to $x^2 + y^2 = 25$ at the point $(3, 4)$.
Give your answer in the form $ax + by + c = 0$.

32) $f(x) = x^2$. For parts a) to c) below, sketch the graphs of $y = f(x)$ and the given transformation.

    a) $y = f(x) + 3$
    b) $y = f(x + 3)$
    c) $y = -f(x)$

Trigonometry and Vectors

These topics are in Section 6 — p.42-50.

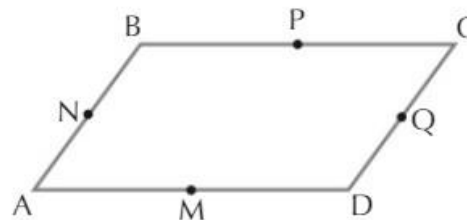**33)** Find the unknowns in each of these triangles. Give your answers to 1 decimal place.
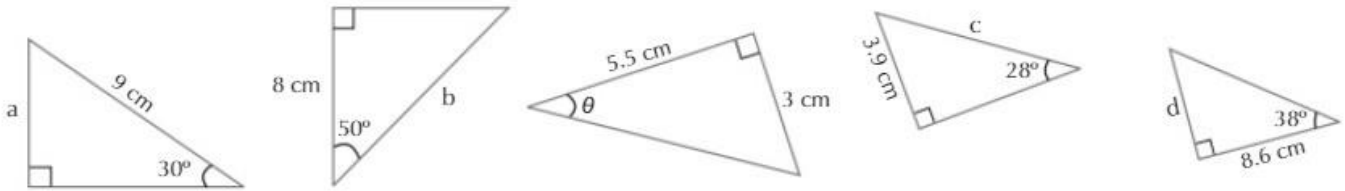
**34)** Find the unknowns in each of these triangles. Give your answers to 1 decimal place.
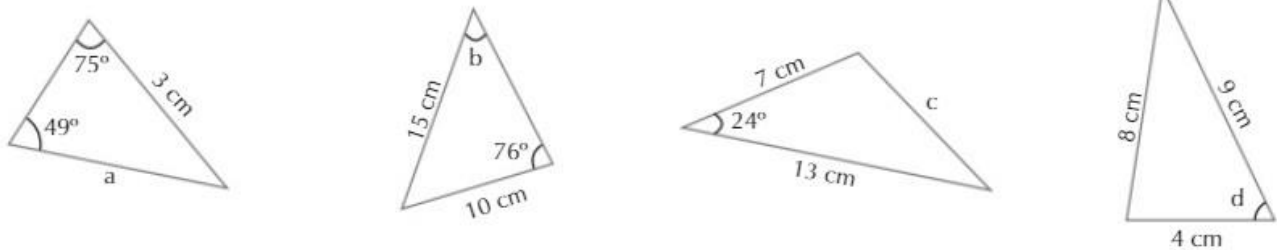
35) ABCD is the parallelogram shown on the right.
M, N, P and Q are the midpoints of the sides.
$\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{BC} = \mathbf{b}$.
Find the following vectors in terms of $\mathbf{a}$ and $\mathbf{b}$.

a) $\overrightarrow{AC}$ b) $\overrightarrow{DQ}$ c) $\overrightarrow{CM}$

d) $\overrightarrow{QP}$ e) $\overrightarrow{MB}$ f) $\overrightarrow{PA}$

36) The diagram shows triangle ABC.
M is the midpoint of $\overrightarrow{AC}$ and N is the midpoint of $\overrightarrow{BC}$.
$\overrightarrow{AM} = 3\mathbf{a} - \mathbf{b}$ and $\overrightarrow{NC} = 2(\mathbf{a} - \mathbf{b})$.
Show that $\overrightarrow{AB}$ and $\overrightarrow{MN}$ are parallel.
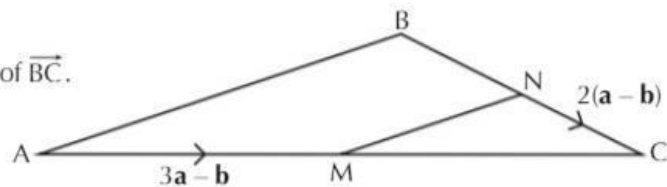
Sampling and Histograms

You'll find these topics on p.51-53.

37) Describe how a simple random sample of size 20 can be selected from a population of 200.

38) The weights of the chocolate bars in a shop storeroom
are shown in the table and histogram below.

a) Use the information in the table and the
histogram to label the vertical axis.

b) Use the histogram to complete the table.

c) Use the table to add the missing bar to the histogram.

Weight (w, in grams)	Frequency
$0 < w \leq 100$	50
$100 < w \leq 150$	100
$150 < w \leq 200$	150
$200 < w \leq 250$	
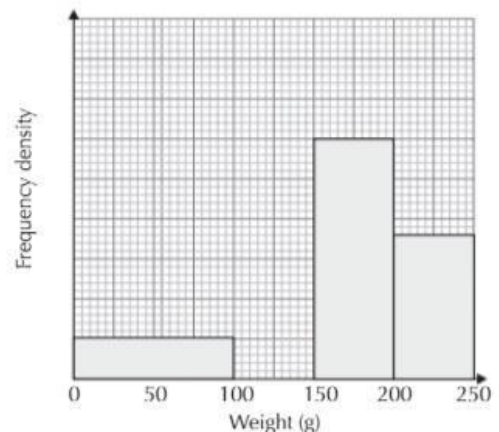
Averages and Cumulative Frequency

Averages are covered on p.54-55.

39) Find the mean, median and mode(s) of these numbers:

| 5 | 3 | −2 | 0 | −3 | 2 | 1 | 1 | 4 | 2 | 6 | 11 | −4 |

40) The table shows the journey times between home and school for 60 students.

a) Write down the modal class.

b) Which group contains the median?

c) Estimate the mean value.

d) Draw a cumulative frequency graph for the data in the table.

Time (m minutes)	Frequency
$5 < m \leq 10$	4
$10 < m \leq 15$	25
$15 < m \leq 20$	18
$20 < m \leq 25$	8
$25 < m \leq 30$	5

41) Using this cumulative frequency graph, find the:
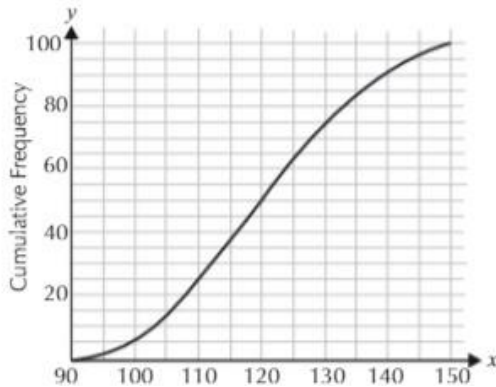
a) median

b) lower quartile

c) upper quartile

d) interquartile range

You can learn about cumulative frequency on p.56.

Probability and Tree Diagrams

You can learn about these topics on p.57-60.

42) Lewis asked 50 people if they like mashed potatoes (M) and roast potatoes (R). The Venn diagram shows the results.

A person is chosen at random.
Find the probability that they:

a) like mashed potatoes

b) like neither mashed nor roast potatoes

c) like both types of potatoes

d) don't like roast potatoes

e) don't like mashed potatoes
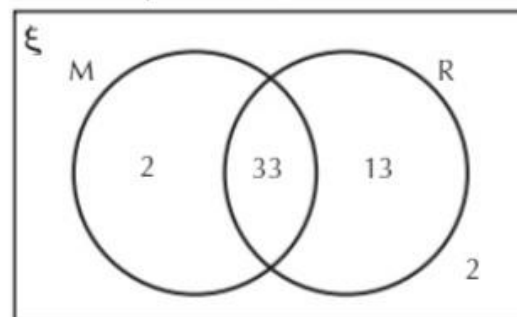
43) Mona's purse contains two £5 notes, four £10 notes and three £20 notes. It also contains five 20p coins, four 50p coins and three £1 coins.

a) Mona picks one note and one coin at random from her purse. Find the probability that she picks a £5 note and a 20p coin.

b) Mona picks two coins at random without replacement. Use a tree diagram to find the probability she picks a 50p coin and a £1 coin.

1   4, –10, 205 and 0 are integers.

2    $5.\dot{9}$, $\frac{1}{5}$, –6, $\sqrt{4}$, 13.978 and 2.1 are rational.

   π and $\sqrt{7}$ are irrational.

3 a) $\frac{2}{15}$

   b) $\frac{1}{4}$

   c) $\frac{11}{12}$

   d) $\frac{51}{35}$

4 a) x^9
   b) $2y^2$
   c) 1
   d) $32n^{10}$

5    $\frac{1}{25}$

6 a) $\frac{9}{16}$

   b) 4

   c) 4

   d) $\frac{1}{6}$

7 a) $x^2 - 2x - 24$
   b) $x^2 + 10x + 25$
   c) $2x^2 + 5x - 3$
   d) $x^3 + 2x^2 - 19x - 20$

8 a) $5(x + 4)$

   b) $3a(1 + 4b)$

   c) $(x + 2)(x - 2)$

   d) $9(x + 2)(x - 2)$

   e) $(x + \sqrt{5})(x - \sqrt{5})$

9 a) $\sqrt{6}$
   b) 5
   c) $\sqrt{5}$
   d) $4\sqrt{3}$
   e) $8 + 2\sqrt{7}$

10 a) $\frac{3\sqrt{2}}{2}$

   b) $\frac{\sqrt{10}}{4}$

   c) $\frac{6 - 2\sqrt{6}}{3}$

   d) $\frac{\sqrt{2} + \sqrt{10}}{-4}$

11 a) $x = 2$
   b) $x = 10$
   c) $x = -5$
   d) $x = -3$ or $x = 3$

12 a) $x = \frac{y - c}{m}$

   b) $x = \frac{5y - 2}{3}$

   c) $x = \pm\sqrt{\frac{y - 1}{2z}}$

   d) $x = \frac{2y + 1}{y - 3}$

13 a) $x = 2$ or $x = 1$
   b) $x = -5$ or $x = -1$
   c) $x = 2.5$ or $x = -1$
   d) $x = \frac{4}{3}$ or $x = 3$

14 a) $x = 2.32$ or $x = -4.32$
   b) $x = 2.69$ or $x = -0.19$

15 a) $x = 2 + \sqrt{6}$ or $x = 2 - \sqrt{6}$

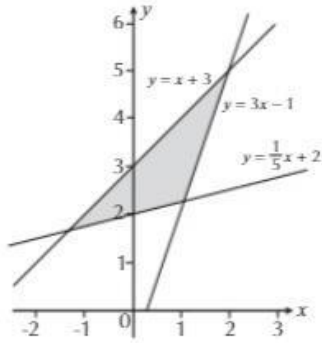   b) $x = -1 + \frac{3}{\sqrt{2}}$ or $x = -1 - \frac{3}{\sqrt{2}}$

16 a) $x^2 + 6x + 8 = (x + 3)^2 - 1$
   b)

(–4, 0)   (–2, 0)   (–3, –1)
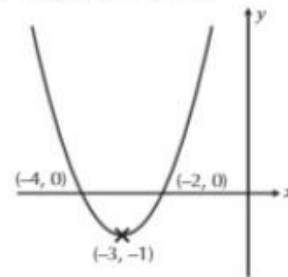
17 a) $3ab^2$

   b) $\frac{1}{8y}$

   c) $\frac{x - 4}{x - 5}$

18 a) $6ab$

   b) $\frac{x - 1}{3}$

   c) 27

   d) $\frac{5x^2 - x - 3}{x^2(x + 1)}$

19 a) $x \le -1$
   b) $x < 8$
   c) $-3 < x < 3$
   d) $x \le -2$ or $x \ge 2$
   e) $x \le 1$ or $x \ge 5$

20

21 a) $x = 2, y = -2$
   b) $x = 11, y = 16$
   c) $x = -3, y = 12$ or $x = 5, y = 28$
   d) $x = 0, y = -2$ or $x = 3, y = 4$

22 Take three consecutive odd numbers:
    $2n + 1$, $2n + 3$ and $2n + 5$, where n is an integer.
    $2n + 1 + 2n + 3 + 2n + 5 = 6n + 9 = 3(2n + 3)$
    The sum of three consecutive odd numbers can be
    written as $3x$, where $x = 2n + 3$.
    Therefore it is a multiple of 3.

23 E.g. Let $x = 3$ and $y = -1$. So $xy = -3 \Rightarrow xy < y$.
    So Naveen is wrong.

24 a) 3
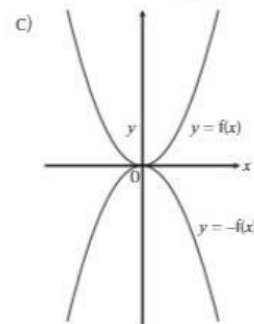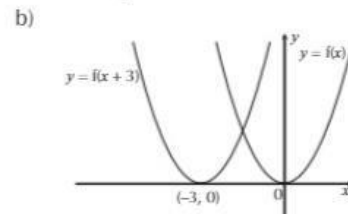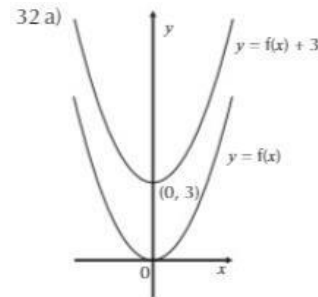   b) $fg(x) = \dfrac{x + 2}{3}$
   c) $f^{-1}(x) = 3x - 5$

25 Gradient $= -0.5$, y-intercept $= 2$

26 a) $y = 2x - 8$
   b) $3\sqrt{5}$

27 a) $y = 2x - 4$
   b) $y = -\dfrac{1}{2}x + 2$

28

29 a)

b)

c)
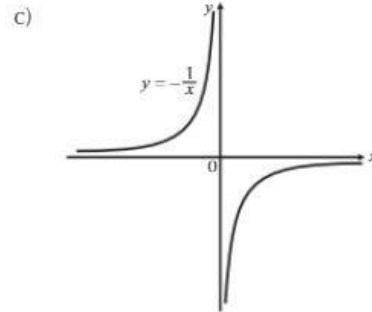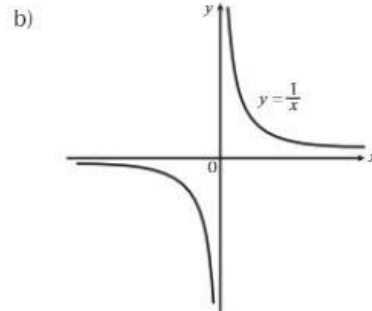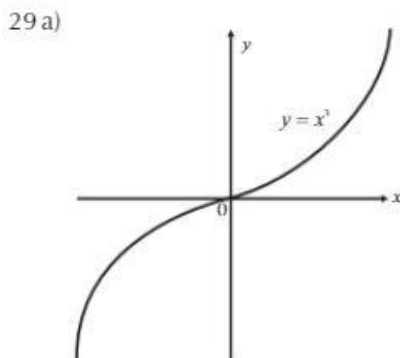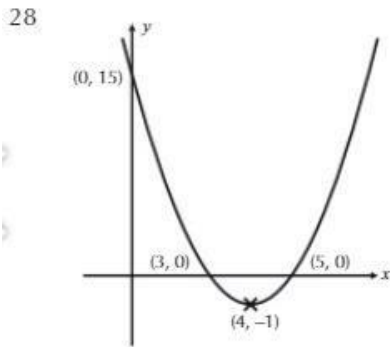
30 $m = 8, n = 2$

31 $3x + 4y - 25 = 0$

32 a)

b)

c)

33 $a = 4.5$ cm, $b = 12.4$ cm, $\theta = 28.6°$,
    $c = 8.3$ cm, $d = 6.7$ cm

34 $a = 3.8$ cm, $b = 40.3°$, $c = 7.2$ cm, $d = 62.7°$

35 a) $\mathbf{a} + \mathbf{b}$

b) $\frac{1}{2}\mathbf{a}$

c) $-\mathbf{a} - \frac{1}{2}\mathbf{b}$

d) $\frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}$

e) $\mathbf{a} - \frac{1}{2}\mathbf{b}$

f) $-\mathbf{a} - \frac{1}{2}\mathbf{b}$

36 $\vec{AB} = 2(3\mathbf{a} - \mathbf{b}) - 2(2(\mathbf{a} - \mathbf{b}))$
   $= 6\mathbf{a} - 2\mathbf{b} - 4\mathbf{a} + 4\mathbf{b} = 2\mathbf{a} + 2\mathbf{b} = 2(\mathbf{a} + \mathbf{b})$

   $\vec{MN} = 3\mathbf{a} - \mathbf{b} - 2(\mathbf{a} - \mathbf{b}) = 3\mathbf{a} - \mathbf{b} - 2\mathbf{a} + 2\mathbf{b} = \mathbf{a} + \mathbf{b}$

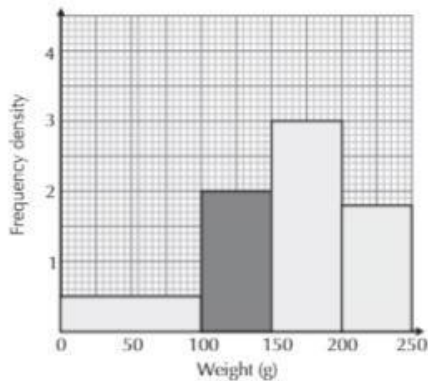 $\vec{AB} = 2\vec{MN} \Rightarrow \vec{AB}$ and $\vec{MN}$ are parallel.

37 First assign a unique number between 1 and 200 to every member of the population. Then create a list of 20 random numbers between 1 and 200. Finally, match the random numbers to members of the population.

38 a) See histogram in part c).

b)

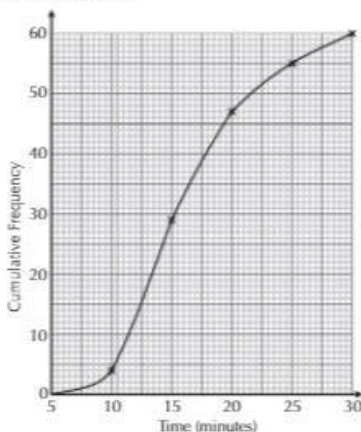| Weight (w, in grams)	Frequency
$0 < w \leq 100$	50
$100 < w \leq 150$	100
$150 < w \leq 200$	150
$200 < w \leq 250$	**90**

c)

39 mean = 2, median = 2, mode = 1 and 2

40 a) $10 < m \leq 15$

b) $15 < m \leq 20$

c) 16.25 minutes

d)

41 a) 120

b) 110

c) 130

d) 20

42 a) $\frac{7}{10}$ (or 0.7)
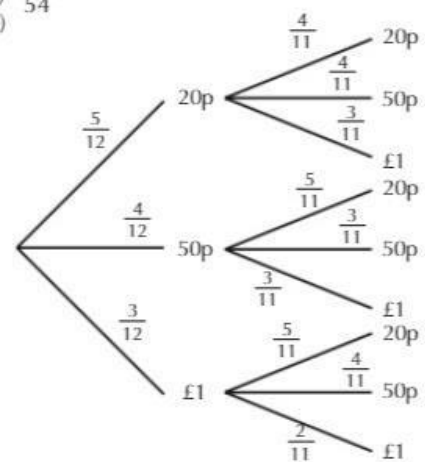
b) $\frac{1}{25}$ (or 0.04)

c) $\frac{33}{50}$ (or 0.66)

d) $\frac{2}{25}$ (or 0.08)

e) $\frac{3}{10}$ (or 0.3)

43 a) $\frac{5}{54}$

b)

Probability of a 50p coin and a £1 coin $= \frac{2}{11}$

Skill	Pages from "Head Start to A-Level Maths"	SparxMaths Tasks	A-Level Revision website
Number			
Types of number	6		
Fractions	7		
Basic Algebra			
Laws of indices	8-9	U662	Topic 02 - Indices
Multiplying out brackets	10	U768, U606	Topic 05 - Algebra
Factorising	11		
Surds	12-13	U499, U707, U281	Topic 03 - Surds
Solving equations	14		
Rearranging formulae	15	U181 U191	Topic 01 – Rearranging Formulae
Quadratic Equations			
Factorising quadratics	16-17	U178, U858 U228	Topic 04 – Solving Quadratics by Factorisation
The quadratic formula	18	U665 U150	
Completing the square	19-21	U589	Topic 06 – Completing the Square
More Algebra			
Algebraic fractions	22	U437 U685 U457 U824 U960	Topic 04 – Solving Quadratics by Factorisation Topic 10 – Algebraic Fractions
Inequalities	24	U759 U133	Topic 07 - Inequalities
Graphical inequalities	26	U747	
Simultaneous equations	28	U760 U757	
Proof	30	U582	
Functions	32	U996 U448	
Graphs			
Straight lines	34	U315 U477 U848 U669 U377	Topic 08 – Straight Lines
Parallel and perpendicular lines	36	U898	Topic 09 – Further Straight Lines
Quadratic graphs	37	U769 U601	
Harder graphs	38	U980 U229 U593 U567	
Graph transformations	40	U455	
Trigonometry and Vectors			
Trigonometry – sin, cos, tan	42	U283 U545 U170	
Trigonometry – graphs	44		
The sine and cosine rules	47	U952 U591	
Vectors	49	U781	

Sampling	51	U162	
Data basics	52	U312	
Histograms	53	U983 U814	
Averages	54	U717	
Cumulative frequency	56	U182 U642	
Probability	57	U476 U699	
Laws of probability	59	U683	
Tree diagrams	60	U558 U729 U806	